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HIGGS PRODUCTION IN GLUON FUSION TO $\mathcal{O}(\alpha_s^4)$*

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The calculation of the NNLO QCD corrections to the partonic process $gg \to H$ is outlined. For the coupling of the Higgs boson to the gluons we use an effective Lagrangian in the limit of a heavy top quark. The focus is on the evaluation of the virtual two-loop corrections. It is shown that the leading pole terms are in agreement with the general formula by Catani.

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1. Introduction

Gluon fusion will be the dominant production mechanism of a Standard Model Higgs boson at the LHC. For a Higgs mass between 100 and 200 GeV, gluon fusion exceeds all other production channels by a factor ranging from five to eight[1]. QCD radiative corrections to this process are found to be more than 50% at NLO[2]. The evaluation of the NNLO corrections ($\mathcal{O}(\alpha_s^4)$) is therefore highly desirable.

The first step towards the full result was taken recently[3], when the virtual NNLO corrections were evaluated. Although infra-red divergent, the result could be used to deduce an expectation of the size of the full NNLO corrections. This estimate turns out to be roughly 10–20%, indicating that the perturbative expansion is valid.

There are several more steps to be taken in order to arrive at a prediction of the inclusive cross section at NNLO. First, single and double real radiation have to be added to the virtual corrections and all terms must be renormalized. After factorization of the soft singularities into the splitting functions, this leads to an IR and UV finite result. Finally, the latter has to be folded with the corresponding parton distribution functions at NNLO whose evaluation is still awaited.

In the following we will focus mainly on the virtual corrections, in particular on the comparison of the leading poles in $\epsilon = (4 - D)/2$ with a general result by Catani[4] (D is the space-time dimension which is used for the regularization of the integrals). For more details on the actual calculation and the results we refer to Ref.[3].

2. Virtual two-loop corrections

It has been shown[2] that the limit of a heavy top quark is well justified in the process $gg \to H$. Technically this means that the top quark loop that mediates the

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coupling of the gluons to the Higgs boson can be replaced by an effective vertex. In this way, the NNLO contribution is represented by two-loop vertex diagrams with two massless on-shell legs. Such diagrams can be evaluated by mapping them onto three-loop massless propagator diagrams[5] and reducing them through the well-known integration-by-parts algorithm[6].

We write the virtual contribution to the cross section for the process $gg \to H$ as[a]

$$\begin{aligned}
\sigma_{\rm virt} &= \frac{4\pi}{v^2}\left(\frac{C_1(\alpha_s)}{1-\beta(\alpha_s)/\epsilon}\right)^2 \frac{\delta(1-z)}{256(1-\epsilon)}\left|1+\frac{\alpha_s^{\rm B}}{\pi}a^{(1)}+\left(\frac{\alpha_s^{\rm B}}{\pi}\right)^2 a^{(2)}+\mathcal{O}(\alpha_s^3)\right|^2 = \\
&= \frac{4\pi}{v^2}\delta(1-z)\,\frac{C_1^2(\alpha_s)}{256(1-\epsilon)}\left|1+\frac{\alpha_s}{\pi}a_{\rm ren}^{(1)}+\left(\frac{\alpha_s}{\pi}\right)^2 a_{\rm ren}^{(2)}+\mathcal{O}(\alpha_s^3)\right|^2 , \\
\beta(\alpha_s) &= -\frac{\alpha_s}{\pi}\beta_0-\left(\frac{\alpha_s}{\pi}\right)^2\beta_1+\mathcal{O}(\alpha_s^3)\,, \quad \beta_0=\frac{33-2n_l}{12}\,, \quad \beta_1=\frac{153-19n_l}{24}\,.
\end{aligned} \tag{1}$$

$C_1(\alpha_s)$ is the coefficient function for the ggH vertex and contains the residual logarithmic top mass dependence[7] ($l_t = \ln(\mu^2/M_t^2)$, with M_t the on-shell top quark mass, $n_l = 5$ denotes the number of light quark flavors, and $\alpha_s = \alpha_s^{(5)}(\mu^2)$ is the running coupling for five active flavors)

$$C_1(\alpha_s) = -\frac{\alpha_s}{3\pi}\left\{1+\frac{11}{4}\frac{\alpha_s}{\pi}+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{2777}{288}+\frac{19}{16}l_t+n_l\left(-\frac{67}{96}+\frac{1}{3}l_t\right)\right]\right\}+\mathcal{O}(\alpha_s^4)\,.$$

The coefficient $a_{\rm ren}^{(1)}$ up to $\mathcal{O}(\epsilon^0)$ reads[2] ($\zeta_2 = \pi^2/6$)

$$a_{\rm ren}^{(1)} = e^{i\pi\epsilon}\left(\frac{\mu^2}{M_{\rm H}^2}\right)^\epsilon\left[-\frac{3}{2\epsilon^2}+\frac{3}{4}\zeta_2\right]+\frac{1}{\epsilon}\left(-\frac{11}{4}+\frac{1}{6}n_l\right)\,. \tag{2}$$

According to Catani's general result[4], the UV-renormalized NNLO contribution can be cast into the following form:

$$a_{\rm ren}^{(2)} = \frac{1}{2}\boldsymbol{I}^{(1)}(\epsilon)\,a_{\rm ren}^{(1)}+\frac{1}{4}\boldsymbol{I}^{(2)}(\epsilon)+a_{\rm fin}^{(2)}\,, \tag{3}$$

where $a_{\rm fin}^{(2)}$ is finite as $\epsilon \to 0$. In our case of two incoming gluons, the operator $\boldsymbol{I}^{(1)}(\epsilon)$ reads[b] ($\gamma_{\rm E} = 0.577216\ldots$)

$$\boldsymbol{I}^{(1)}(\epsilon) = -\left(\frac{\mu^2}{M_{\rm H}^2}\right)^\epsilon \frac{e^{i\pi\epsilon}e^{\gamma_{\rm E}\epsilon}}{\Gamma(1-\epsilon)}\left[\frac{3}{\epsilon^2}+\frac{1}{\epsilon}\left(\frac{11}{2}-\frac{1}{3}n_l\right)\right]\,. \tag{4}$$

The expression for $\boldsymbol{I}^{(2)}(\epsilon)$ is

$$\begin{aligned}
\boldsymbol{I}^{(2)}(\epsilon) = &-\frac{1}{2}\boldsymbol{I}^{(1)}(\epsilon)\left(\boldsymbol{I}^{(1)}(\epsilon)+\frac{4}{\epsilon}\beta_0\right)+ \\
&+\frac{e^{-\epsilon\gamma_{\rm E}}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{2}{\epsilon}\beta_0+\frac{67}{6}-3\zeta_2-\frac{5}{9}n_l\right)\boldsymbol{I}^{(1)}(2\epsilon)+\boldsymbol{H}^{(2)}(\epsilon)\,.
\end{aligned} \tag{5}$$

[a] Note that Eq. (9) of Ref.[3] erroneously has an additional factor of $M_{\rm H}^2$.

[b] Note that due to a misprint ($\lambda_{ij} = +1$ instead of $\lambda_{ij} = -1$) the original formula[4] results in a different sign of the unitary phase. A clarifying conversation on this issue with S. Catani and S. Dittmaier is acknowledged.

$\boldsymbol{H}^{(2)}(\epsilon)$ contains only single poles in ϵ. Thus, Eqs. (2)–(5) determine the poles of order $1/\epsilon^4$, $1/\epsilon^3$, and $1/\epsilon^2$ of the NNLO amplitude $a^{(2)}_{\rm ren}$. They can now be compared with the explicit evaluation of the Feynman diagrams at NNLO[3], and full agreement is found.[c] This provides a non-trivial check on the results of Ref.[3]. From the latter, one may now extract the undetermined pieces $\boldsymbol{H}^{(2)}(\epsilon)$ of Eq. (5) and $a^{(2)}_{\rm fin}$ of Eq. (3). Their values depend on whether or not one keeps the higher order terms in ϵ when expanding the exponentials and the Γ-functions in Eqs. (2)–(5). Thus, in order to prevent confusion, we refrain from giving explicit expressions for $\boldsymbol{H}^{(2)}(\epsilon)$ and $a^{(2)}_{\rm fin}$. Instead, we advise the interested reader to use the results of Ref.[3].

3. Real radiation

Let us give a brief description of the contributions coming from the real emission of quarks and gluons. The divergences of the virtual two-loop corrections will be canceled by the soft contributions of the following processes: $gg \to Hg$ to one-loop order, and $gg \to Hgg$, $gg \to Hq\bar{q}$ at tree level (assuming the effective ggH vertex). After adding these contributions to the virtual corrections, there are still infra-red singularities left. These have to be absorbed by the mass factorization procedure, using the Altarelli-Parisi splitting functions at NNLO.

Explicit results for the full partonic calculation will be given elsewhere[8]. For the contribution $\propto \delta(1 - s/M_{\rm H}^2)$ (s is the cms energy of the gg-system) we find a NNLO correction of order 10%, in agreement with the numerical estimate of Ref.[3].

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[c] It is important to note that the prescriptions for UV-renormalization of Ref.[4] and Ref.[3] differ in the treatment of terms $\propto \gamma_{\rm E}$ and $\propto \ln 4\pi$. For convenience, we apply the former prescription here. For infra-red finite quantities, the two prescriptions are equivalent up to terms of order ϵ.